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Micromechanics modeling of composite with ductile matrix and shape memory alloy reinforcement

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Abstract

A quantitative micromechanics-based analysis on the role of microstructure and constituent properties in the overall behavior of shape memory alloy (SMA) composite is carried out in the present work. The composite consists of ductile matrix and SMA second phase inclusions. The macroscopic constitutive relations of the composite are established by using self-consistent approach where the micro–macro correlation is realized by volume averaging and by introducing the concept of stress and strain concentration tensors. In this micromechanics modeling, the internal stress and strain in both matrix and SMA and their evolution are derived as function of externally applied thermomechanical loading as well as the degree of phase transformation in SMA. As an application of the present theory in the microstructural design of this novel composite, the constitutive response of composites with spherical SMA particulate embedded in two different elastoplastic matrixes under uniaxial tension is calculated. The obtained results demonstrate several interesting deformation features of the new composite, which are expected to have potential applications in the future. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Micromechanics modeling; Shape memory alloy composite; Ductile matrix

1. Introduction

Shape memory alloys (SMAs) have recently attracted interest in the field of composite materials and have been proposed as sensors and large strain actuators for use in intelligent composites and structures. This is because SMAs have native ability to undergo reversible thermoelastic martensitic phase transformation under external thermomechanical loading. SMAs, when in the form of wires, short

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fibers, particulates or thin films, can be embedded into or hybridized with a host material to form an SMA composite. The design concept of an 'intelligent SMA composite' has been recently proposed. Its aim is to realize smart composites by 'intelligent' distribution of SMA within the matrix material and control the thermomechanical behavior of SMA through heating and cooling. So far, some fundamental techniques in manufacturing and metallurgical treatment have been developed, and prototype SMA composite materials have been manufactured and tested. The potential applications of embedded SMAs include controlling external shape, stiffness, damage, vibration, buckling and damping properties of the composites (Birman, 1997).

Increasing effort has been made in the manufacturing and processing of the SMA composites by embedding SMA into either elastic or ductile matrix (see Rogers, 1990; Paine and Rogers, 1991; Bidaux et al., 1995; Hamada et al., 1997; Lee et al., 1997) and some research has been done to understand and model the constitutive behavior of SMA composite with elastic matrix (Boyd and Lagoudas, 1994; Lagoudas et al., 1994; Sottos and Kline, 1996; Stalmans et al., 1997). In these works, quantitative understanding of the interaction between the embedded SMA and the matrix is one of the critical factors in the microstructure design of the composites. In developing such materials it is a key issue to have a precise correlation among the evolution of the internal stress and strain state, the externally applied thermomechanical loading, microstructure and the constituent properties. For example, in order to create a favorable residual stress field in both matrix and SMA and also a desired macroscopic behavior, one has to select the constitutive and microstructure parameters properly so that an optimum microstructure design can be realized. To date, most prototype SMA composites are designed and fabricated based on an empirical approach or simple calculations. Systematic and quantitative investigation on how to make 'intelligent' use of thermomechanical properties of SMAs and matrix materials by using a microstructure-based micromechanics approach has not been available in the literature. Without doubt, such an investigation will be more comprehensive and be helpful for the composite microstructure design.

This paper aims to establish a micromechanics-based modeling on the constitutive behavior of the SMA composite with elastoplastic matrix. In Section 2 the constitutive behavior of the elastoplastic matrix and SMA are first described. A generalized overall-local relationship is established in Section 3. This type of micro–macro correlation is realized by volume averaging and by introducing the concepts of stress and strain concentration tensors. The concentration tensors are determined by a self-consistent approach. The constitutive relations for different microstructures under general loading conditions are formulated. In Section 4 the constitutive model is applied to calculate the constitutive response and internal stress and strain evolution of two different elastoplastic matrixes composites respectively. The obtained results are discussed and distinct features of this type of composites are identified. As a first step in the investigation, the thermal expansion of two phases is neglected and the focus is on the mechanical aspects of the problem only.

2. The constitutive behavior of matrix and SMAs

The two-phase composite considered in this paper consists of an elastoplastic ductile matrix phase (with a superscript 'M' in all related quantities throughout the paper) and an elastoplastic second phase SMA inclusions (all related quantities with a superscript 'I' throughout the paper). In SMA the 'plasticity' is due to martensitic transformation only. Perfect bonding is assumed between the two phases. The SMA particulates may have a given distribution in the matrix (such as unidirectional distribution). Fig. 1 illustrates the Representative Volume Element (RVE) of the composite. The volume fraction of the SMA is denoted by f. Throughout this paper, Σ_{ij} and $\dot{\Sigma}_{ij}$ denote macroscopic externally applied stress tensor and its rate, E_{ij} and $\dot{E}_{ij}$ denote macroscopic externally applied strain tensor and its

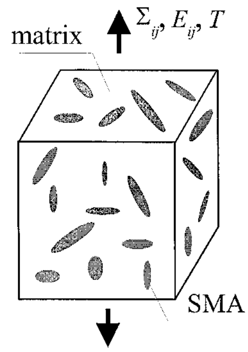

Fig. 1. Schematic of the constitutive element of the composite.

rate, σ_{ij} and $\dot{\sigma}_{ij}$ denote microscopic stress tensor and its rate, ε_{ij} and $\dot{\varepsilon}_{ij}$ denote microscopic (or local) total strain tensor and its rate and $\varepsilon_{ij}^{\mathrm{tr}}$ and $\dot{\varepsilon}_{ij}^{\mathrm{tr}}$ denote transformation strain tensor and its rate in SMA.

2.1. Constitutive behavior of matrix

The ductile matrix materials considered here are assumed to be homogeneous and their response are time independent. The rate form of the constitutive relations under small strain condition can be described by

$$\dot{\sigma}_{ij} = C_{ijkl}^{\mathrm{M}}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^{\mathrm{p}}\right) \tag{1}$$

where C_{ijkl}^{M} is elastic tensor, $\dot{\varepsilon}_{ij}^{\mathrm{p}}$ is plastic strain rate. By using the tangent modulus tensor l_{ijkl}^{M}, eqn (1) can also be expressed as

$$\dot{\sigma}_{ij} = l_{ijkl}^{\mathrm{M}}\dot{\varepsilon}_{kl} \tag{2}$$

The plastic strain rate can be determined through plasticity theory. For the ductile materials described by the Ludwick type yielding function

$$\sigma_{\mathrm{e}} = \sigma_{\mathrm{e}}^{y} + h^{\mathrm{M}}\left(\varepsilon_{\mathrm{e}}^{\mathrm{p}}\right)^{n} \tag{3}$$

the plastic strain rate can be derived as

$$\dot{\varepsilon}_{ij}^{\mathrm{p}} = \eta\frac{9\sigma_{ij}^{d}\sigma_{kl}^{d}}{4H^{\mathrm{M}}\sigma_{\mathrm{e}}^{2}}\dot{\sigma}_{kl} \tag{4}$$

and if elastic property of material is isotropic, the tangent modulus tensor l_{ijkl}^{M} can be expressed as

$$l_{ijkl}^{\mathrm{M}} = C_{ijkl}^{\mathrm{M}} - \eta\frac{9\left(\mu^{\mathrm{M}}\right)^{2}\sigma_{ij}^{\mathrm{d}}\sigma_{kl}^{\mathrm{d}}}{\left(H^{\mathrm{M}} + 3\mu^{\mathrm{M}}\right)\sigma_{\mathrm{e}}^{2}} \tag{5}$$

where

$$\sigma_{\mathrm{e}}\left(\sigma_{\mathrm{e}} = \left(\frac{3}{2}\sigma_{ij}^{\mathrm{d}}\sigma_{ij}^{\mathrm{d}}\right)^{\frac{1}{2}}, \quad \sigma_{ij}^{\mathrm{d}} = \sigma_{ij} - \frac{\sigma_{kk}}{3}\delta_{ij}\right)$$

is the Von Mises equivalent stress, σ_{e}^{y} is the initial Von Mises yield stress, $\varepsilon_{\mathrm{e}}^{\mathrm{p}}(\varepsilon_{\mathrm{e}}^{\mathrm{p}} = \int \dot{\varepsilon}_{\mathrm{e}}^{\mathrm{p}} \mathrm{d}t$, $\dot{\varepsilon}_{\mathrm{e}}^{\mathrm{p}} = (2/3\, \dot{\varepsilon}_{ij}^{\mathrm{p}} \dot{\varepsilon}_{ij}^{\mathrm{p}})^{1/2})$ is the accumulated effective plastic strain, both h^{M} and n are the hardening parameters, μ^{M} is the elastic shear modulus, and η is the loading parameter ($\eta = 0$ for elastic unloading and $\eta = 1$ for plastic loading), $H^{\mathrm{M}} = \mathrm{d}\sigma_{\mathrm{e}}/\mathrm{d}\varepsilon_{\mathrm{e}}^{\mathrm{p}}$.

2.2. Constitutive behavior of shape memory alloys

The constitutive relations of SMA can generally be given by

$$\sigma_{ij} = l_{ijkl}^{\mathrm{I}}\left(\varepsilon_{kl} - \varepsilon_{kl}^{\mathrm{tr}}\right) \tag{6}$$

where l_{ijkl}^{I} is the elastic modulus tensor, $\varepsilon^{\mathrm{tr}}$ is the total transformation strain of SMA. For most SMA materials, l_{ijkl}^{I} is the function of temperature T and martensite volume fraction ζ in SMA. Then rate form of eqn (6) can be written as

$$\dot{\sigma}_{ij} = l_{ijkl}^{\mathrm{I}}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^{\mathrm{tr}} + \dot{Q}_{kl}\right) \tag{7}$$

where

$$\dot{Q}_{ij} = \left(l_{ijmn}^{\mathrm{I}}\right)^{-1}\left(\frac{\partial l_{mnkl}^{\mathrm{I}}}{\partial \zeta}\dot{\zeta} + \frac{\partial l_{mnkl}^{\mathrm{I}}}{\partial T}\dot{T}\right)\left(\varepsilon_{kl} - \varepsilon_{kl}^{\mathrm{tr}}\right) \tag{8}$$

The evolution of variables $\dot{\zeta}$ and $\dot{\varepsilon}_{ij}^{\mathrm{tr}}$ can be derived from the constitutive models of specific SMAs (see Section 4). There has been a significant amount of research dedicated to the modeling within the last ten years for several practically used polycrystalline SMAs. The reader is referred to the following for exhaustive description: (1) phenomenological macroscopic constitutive model (Tanaka et al., 1986; Liang and Rogers, 1990; Raniecki et al., 1992; Bekker and Brinson, 1997); (2) constitutive model based on micromechanics mean field theory (Patoor et al., 1988; Sun and Hwang, 1993; Boyd and Lagoudas, 1996; Song et al., 1997; Lu and Weng, 1997); (3) thermoelastic theory of phase transition and its applications to SMA (Falk, 1980; Muller and Xu, 1991; Abeyaratne and Knowles, 1993; Bhattacharya, 1993).

Generally, the rate constitutive equations for both matrix and SMA can be formulated in a unified form

$$\dot{\sigma}_{ij} = l_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^{*}\right) \tag{9}$$

where elastoplastic tangent modulus $l_{ijkl} = l_{ijkl}^{\mathrm{M}}$ and eigen strain $\dot{\varepsilon}_{ij}^{*} = 0$ for matrix, and $l_{ijkl} = l_{ijkl}^{\mathrm{I}}$ and $\dot{\varepsilon}_{ij}^{*} = \dot{\varepsilon}_{ij}^{\mathrm{tr}} - \dot{Q}_{ij}$ for SMA.

3. Micromechanics constitutive modeling of the composite

From a micromechanics point of view, the modeling of heterogeneous materials is extensively based on the averaging operations using the original Eshelby's solution of the ellipsoidal inhomogeneity (Eshelby, 1957). The most prominent model is the self-consistent approximation where the interactions among various kinds of inhomogeneities of finite volume concentrations are approximately taken into account by embedding the inhomogeneities in a medium with effective thermomechanical properties. Originally, the self-consistent approximation has been introduced to describe the elastoplastic behavior of polycrystalline materials with two different approaches: (1) the Kroner (1958a, 1958b, 1961) and

Budiansky and Wu (1962) approach (KBW approach); in this approach the elementary problem of inclusion–matrix interaction is solved by taking the difference of plastic strain between the inclusion and the surrounding matrix as the Eshelby stress free strain. As a consequence the constraint power of the matrix during the plastic deformation of the inhomogeneity remains constant and therefore the internal stresses are over estimated. (2) The Hill (1965a, 1965b, 1967) approach where the problem is treated incrementally by using the elastoplastic tangent modulus. The constraint power of the matrix depends on the tangent modulus of the matrix and therefore weakens during the plastic deformation. In general, the Hill self-consistent approximation shows a better accuracy (Hutchinson, 1970) in describing the behavior of polycrystalline materials and thus can be used for any incremental calculations. In case of proportional loading, Berveiller and Zaoui (1976) developed a self-consistent approximation by using the secant modulus of the polycrystal to describe the weakening constraint power of the matrix and a good approximation is obtained. The self-consistent approximation has been applied with success to describe the plasticity under complex loading path and elastoplastic behavior at large strain (Beradai et al., 1987; Lipinski and Berveiller, 1989).

For composite materials, the Mori–Tanaka (1973) method has been widely used due to its simplicity in deriving the overall elastic and elastoplastic behavior (Benviniste, 1987; Tandon and Weng, 1988; Weng, 1990). Christensen (1990) made a critical evaluation for different micromechanics models. In present work, the Hill's approach is used to derive the constitutive law of the composite.

3.1. Choice of the independent variables

Once the microstructure and the constitutive law of each constituent are given, one can determine the global behavior of the composite. There are several choices in formulating the macroscopic constitutive relations of the composite. For example, the incremental stress–strain relations can be written either in the form of

$$\dot{\Sigma}_{ij} = \tilde{L}_{ijkl}\dot{E}_{kl} \tag{10}$$

or

$$\dot{\Sigma}_{ij} = C_{ijkl}\left(\dot{E}_{kl} - \dot{E}^{\mathrm{p}}_{kl}\right) \tag{11}$$

where $\tilde{L}_{ijkl}$ is the overall tangent modulus tensor of the composite, C_{ijkl} is the overall elastic tensor. However the calculated result (or the accuracy) strongly depends on the choice of the independent variables. As discussed above, the KBW or the elastic modulus version, eqn (11), will lead to an over estimation of the internal stresses, so here the tangent modulus version, eqn (10), is adopted but is expressed in the form of

$$\dot{\Sigma}_{ij} = L_{ijkl}\left(\dot{E}_{kl} - \dot{E}^{*}_{kl}\right) \tag{12}$$

where L_{ijkl} is the elastoplastic tangent modulus tensor of the composite and E^{*}_{kl} is the macroscopic eigen strain tensor. The purpose of introducing $\dot{E}^{*}_{kl}$ is just to emphasize the role of transformation in the composite behavior. Also, eqn (12) is consistent in form with the local constitutive law, eqn (9), for both SMA and matrix. The dependence of $\dot{E}^{*}_{kl}$ on $\dot{\varepsilon}^{*}_{kl}$ will be given in the following.

3.2. Global-local relationships and determination of concentration tensor

For heterogeneous material the determination of the overall effective properties requires three steps.

(i) The homogenization and localization which link the local stress and strain fields to the overall applied stress or strain by introducing the corresponding concentration tensors. After an adequate averaging scheme, the overall effective properties of the heterogeneous material are given as the functions of these concentration tensors. (ii) The introduction of a homogeneous equivalent matrix to replace the real inhomogeneous matrix. (iii) The evaluation of concentration tensors through a self-consistent method.

3.2.1. Homogenization and localization

The micro–macro or local-global transition generally consists of two aspects: *homogenization* (micro- → macro-) and *localization* (macro- → micro-). It is well known that the overall (or macroscopic) strain and stress rates are respectively the mean values of local strain and stress rates over the RVE of the material.

$$\dot{E}_{ij} = \langle \dot{\varepsilon}_{ij}(r) \rangle_V \tag{13}$$

$$\dot{\Sigma}_{ij} = \langle \dot{\sigma}_{ij}(r) \rangle_V \tag{14}$$

Inversely the local strain and stress rates at any point inside the constitutive element are related to the macroscopic strain and stress rates applied on the boundary of the element by the following forms

$$\dot{\varepsilon}_{ij}(r) = A_{ijkl}(r)\dot{E}_{kl} + a_{ij}(r) \tag{15}$$

$$\dot{\sigma}_{ij}(r) = B_{ijkl}(r)\dot{\Sigma}_{kl} + b_{ij}(r) \tag{16}$$

where A_{ijkl} and B_{ijkl} are respectively the fourth order strain and stress concentration tensors which take into account the heterogeneity of the tangent modulus while $a_{ij}(r)$ and $b_{ij}(r)$ are the corresponding second order tensors to take account of the heterogeneity of the transformation strain rate inside the material. In general $A_{ijkl}(r)$ and $a_{ij}(r)$ (also $B_{ijkl}(r)$ and $b_{ij}(r)$) are not independent to each other because of the coupling effect between plasticity and phase transformation (Cherkaoui et al., 1995; Fischer et al., 1996). Substituting eqns (15) and (16) into eqns (13) and (14) leads to the following equations

$$\dot{E}_{ij} = \langle A_{ijkl}(r) \rangle_V \dot{E}_{kl} + \langle a_{ij}(r) \rangle_V \tag{17}$$

$$\dot{\Sigma}_{ij} = \langle B_{ijkl}(r) \rangle_V \dot{\Sigma}_{kl} + \langle b_{ij}(r) \rangle_V \tag{18}$$

Because the above equations are valid for any $\dot{\Sigma}_{ij}$ and $\dot{E}_{ij}$, we must have the following conditions on $A_{ijkl}(r)$, $a_{ij}(r)$, $B_{ijkl}(r)$ and $b_{ij}(r)$

$$\langle A_{ijkl}(r) \rangle_V = \langle B_{ijkl}(r) \rangle_V = I_{ijkl} \tag{19}$$

$$\langle a_{ij}(r) \rangle_V = \langle b_{ij}(r) \rangle_V = 0 \tag{20}$$

On the other hand, substituting eqns (15) and (16) into eqn (9) and comparing with eqn (12), one can get the following relations among $A_{ijkl}(r)$, $a_{ij}(r)$, $B_{ijkl}(r)$ and $b_{ij}(r)$:

$$B_{ijkl}(r) = l_{ijmn}(r) A_{mnpq}(r) L^{-1}_{pqkl} \tag{21}$$

$$b_{ij}(r) = l_{ijmn}(r)A_{mnpq}(r)\dot{E}^*_{pq} + l_{ijmn}(r)a_{mn}(r) - l_{ijmn}(r)\dot{\varepsilon}^*_{mn}(r) \tag{22}$$

Using $\langle B_{ijkl}\rangle_V = I_{ijkl}$ the average of eqn (21) over V gives the elastoplastic tangent modulus of composite as function of the local elastoplastic tangent modulus and the concentration tensor:

$$L_{ijkl} = \langle l_{ijmn}(r)A_{mnkl}(r)\rangle_V \tag{23}$$

In the same way, $\langle b_{ij}(r)\rangle_V = 0$ gives the macroscopic transformation strain rate $\dot{E}^*_{ij}$ as

$$\dot{E}^*_{ij} = L^{-1}_{ijkl}\langle l_{klmn}(r)\dot{\varepsilon}^*_{mn}(r) - l_{klmn}(r)a_{mn}(r)\rangle_V \tag{24}$$

3.2.2. Transition from a real composite to an equivalent composite

In order to simplify the calculation, an equivalent homogeneous matrix is normally introduced, in almost all self-consistent evaluations of the overall behavior of composites, to replace the real inhomogeneous matrix where the tangent modulus are nonuniform (Fig. 2). The equivalency is built up on the condition that after replacement the overall behavior of the equivalent composite is the same as the original composite. Then the local stress rates become

$$\dot{\sigma}_{ij}(r) = \dot{\sigma}_{ij} = l_{ijkl}(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^*_{kl}) \tag{25}$$

If we denote the volumes of SMA inclusions and ductile matrix by V^{I} and V^{M}, and the volume fraction of the SMA by f, eqn (23) can be written as

$$L_{ijkl} = (1-f)l^{\mathrm{M}}_{ijmn}\langle A_{mnkl}(r)\rangle_{V^{\mathrm{M}}} + fl^{\mathrm{I}}_{ijmn}\langle A_{mnkl}(r)\rangle_{V^{\mathrm{I}}} \tag{26}$$

By using eqns (19) and (20), eqns (26) and (24) can be expressed as

$$L_{ijkl} = l^{\mathrm{M}}_{ijkl} + f\left(l^{\mathrm{I}}_{ijmn} - l^{\mathrm{M}}_{ijmn}\right)\langle A_{mnkl}(r)\rangle_{V^{\mathrm{I}}} \tag{27}$$

$$\dot{E}^*_{ij} = L^{-1}_{ijkl}\left[fl^{\mathrm{I}}_{klmn}\langle\dot{\varepsilon}^*_{mn}(r)\rangle_{V^{\mathrm{I}}} - f\left(l^{\mathrm{I}}_{klmn} - l^{\mathrm{M}}_{klmn}\right)\langle a_{mn}(r)\rangle_{V^{\mathrm{I}}}\right] \tag{28}$$

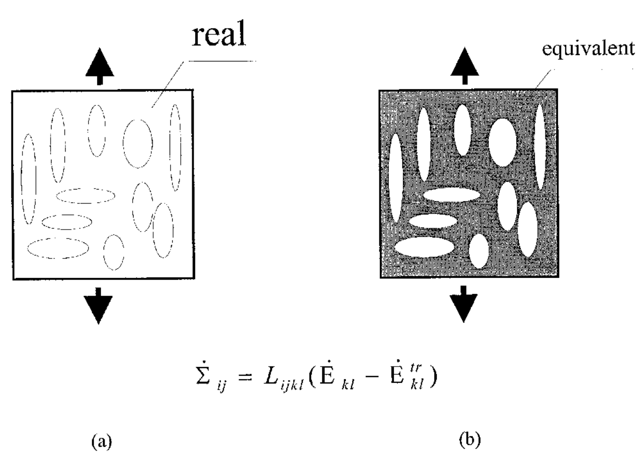

Fig. 2. Schematic of transition from a 'real' (a) composite to an 'equivalent' composite (b).

It should be noted that the relationships established in this section are general and are independent of the choice of micromechanical models used to obtain the concentration tensors in the sub-section 3.2.3.

3.2.3. Self-consistent scheme for the determination of concentration tensors

From eqns (27) and (28), it is apparent that the determination of the overall behavior of the composite requires the average value of strain concentration tensor in the SMA inclusions. By using the self-consistent method the concentration tensors are derived as (see Appendix A for detailed derivation)

$$\langle A_{ijkl}(r)\rangle_{V^{\mathrm{I}}} = \left\langle \left[I_{ijkl} + S_{ijmn}(r) L^{-1}_{mnpq}\left(l^{\mathrm{I}}_{pqkl} - L_{pqkl}\right)\right]^{-1} \right\rangle_{V^{\mathrm{I}}} \tag{29}$$

$$\langle A_{ijkl}(r)\rangle_{V^{\mathrm{M}}} = \left\langle \left[I_{ijkl} + S_{ijmn}(r) L^{-1}_{mnpq}\left(l^{\mathrm{M}}_{pqkl} - L_{pqkl}\right)\right]^{-1} \right\rangle_{V^{\mathrm{M}}} \tag{30}$$

$$\langle a_{ij}(r)\rangle_{V^{\mathrm{I}}} = \left\langle A_{ijkl}(r) S_{klmn}(r) L^{-1}_{mnpq}\left[l^{\mathrm{I}}_{pqrs}\dot{\varepsilon}^{*}_{rs}(r) - L_{pqrs}\dot{E}^{*}_{rs}\right]\right\rangle_{V^{\mathrm{I}}} \tag{31}$$

$$\langle a_{ij}(r)\rangle_{V^{\mathrm{M}}} = -\langle A_{ijkl}(r) S_{klmn}(r)\rangle_{V^{\mathrm{M}}} \dot{E}^{*}_{mn} \tag{32}$$

where $S_{klmn}(r)$ is the Eshelby tensor. It is noted that the above equation has the same form as the one derived for multiphase elastic media by Dvorak and Benveniste (1992).

In case of spherical inclusions or unidirectional distributed spheroids of the identical shape, the Eshelby tensor S_{ijmn} is constant (independent of the position), eqns (29)–(32) are simplified as

$$A^{\mathrm{I}}_{ijkl} = \langle A_{ijkl}(r)\rangle_{V^{\mathrm{I}}} = \left[I_{ijkl} + S_{ijmn} L^{-1}_{mnpq}\left(l^{\mathrm{I}}_{pqkl} - L_{pqkl}\right)\right]^{-1} \tag{33}$$

$$A^{\mathrm{M}}_{ijkl} = \langle A_{ijkl}(r)\rangle_{V^{\mathrm{M}}} = \left[I_{ijkl} + S_{ijmn} L^{-1}_{mnpq}\left(l^{\mathrm{M}}_{pqkl} - L_{pqkl}\right)\right]^{-1} \tag{34}$$

$$a^{\mathrm{I}}_{ij} = A^{\mathrm{I}}_{ijkl} S_{klmn} L^{-1}_{mnpq}\left(l^{\mathrm{I}}_{pqrs}\dot{\varepsilon}^{*}_{rs} - L_{pqrs}\dot{E}^{*}_{rs}\right) \tag{35}$$

$$a^{\mathrm{M}}_{ij} = -A^{\mathrm{M}}_{ijkl} S_{klmn} \dot{E}^{*}_{mn} \tag{36}$$

$$\dot{E}^{*}_{ij} = f L^{-1}_{ijkl} A^{\mathrm{I}}_{mnkl} l^{\mathrm{I}}_{mnab}\dot{\varepsilon}^{*}_{ab} \tag{37}$$

Thus the constitutive framework of the smart composite has been established by the self-consistent approach. By using above relations, the constitutive response of the composite can be directly computed through iteration.

After the matrix begins to yield, its elastoplastic tangent modulus tensor l^{M}_{ijkl} is generally not isotropic. Therefore, the elastoplastic tangent modulus of composite L_{ijkl} will be anisotropic and must be obtained through the iteration of eqns (27) and (33). It must be noticed that Eshelby's tensor S_{ijkl} is related to the L_{ijkl} and then must be calculated by numerical integral. Once the L_{ijkl}, S_{ijkl}, A^{M}_{ijkl}, and A^{I}_{ijkl} are determined, the macroscopic response of composite and the evolution of internal stress, strain and transformation can be evaluated from eqns (15), (16), (25) and (35)–(37).

4. Application to the spherical SMA inclusions

As an application of above model, we consider the case of a composite with spherical second phase SMA. The composite is loaded by uniaxial tension under constant temperature at which the SMA exhibits only superelastic behavior. The constitutive equations of ductile matrix are given by eqns (2)–(5) in Section 2.1. For SMAs, the relation given in eqns (6)–(8) is for the general cases. Most experimental data (Patoor et al., 1988; Muller and Xu, 1991; Lin et al., 1994; 1996; Shaw and Kyriakids, 1995) showed that the slopes of the two elastic branches of the superelastic stress–strain curve of SMAs have not much difference. So normally we can assume that the elastic properties of martensite and parent phase are the same, thus l^{I}_{ijkl} in eqn (6) becomes constant under isothermal loading condition. This assumption is just for the simplicity in calculation. It would not affect the present micromechanics model if the elastic tensors of the two phases were quite different (see the derivation of Section 3). About the thermal behavior of the SMA composite, generally it is a complicated process that depends on the heat transfer conditions as well as the loading rates and is out of the scope of this paper. However it is helpful to notice that the change in temperature comes from two major sources. One is the external applied heating or cooling, another is the self-heating and self-cooling due to the transformation latent heat (Shaw and Kyriakides, 1997). The response of SMA composite under uniform temperature change was studied recently by the authors (Song et al., 1999).

The evolution of transformation is macroscopically similar to that of plastic strain in perfect plasticity of metals (Lin et al., 1994; 1996; Shaw and Kyriakides, 1995). So the transformation condition can be expressed as

$$\sigma^{\mathrm{I}}_{\mathrm{e}} = \begin{cases} \sigma^{y(\mathrm{I})}_{\mathrm{e}}(T) & \text{for forward transformation} \\ \sigma^{y\mathrm{re}(\mathrm{I})}_{\mathrm{e}}(T) & \text{for reverse transformation} \end{cases} \tag{38}$$

where $\sigma^{\mathrm{I}}_{\mathrm{e}}$ is Von Mises equivalent stress in SMA. The transformation strain rate $\dot{\varepsilon}^{\mathrm{tr}}_{ij}$ can be expressed as (Boyd and Lagoudas, 1994; Song et al., 1998)

$$\dot{\varepsilon}^{\mathrm{tr}}_{ij} = \dot{\varepsilon}^{*} = \begin{cases} \mathrm{d}\lambda\sigma^{\mathrm{d(I)}}_{ij} & \text{for forward transformation} \\ \mathrm{d}\lambda^{\mathrm{re}}\varepsilon^{\mathrm{tr}}_{ij} & \text{for reverse transformation} \end{cases} \tag{39}$$

where $\sigma^{\mathrm{d(I)}}_{ij}$ is deviatoric stress in SMA, $\mathrm{d}\lambda$ and $\mathrm{d}\lambda^{\mathrm{re}}$ are proportional factors and must be determined by the consistency condition below. Rewriting formulae (37) and (35) by the simple form

$$\dot{E}^{*}_{ij} = D_{ijkl}\dot{\varepsilon}^{\mathrm{tr}}_{kl} \tag{40}$$

$$a^{\mathrm{I}}_{ij} = F^{\mathrm{I}}_{ijkl}\dot{\varepsilon}^{\mathrm{tr}}_{kl} \tag{41}$$

at each loading step $\dot{\Sigma}_{ij}$, the averaged stress increment in SMA can be obtained from eqns (9), (15) and (41) as

$$\dot{\sigma}^{\mathrm{I}}_{ij} = l^{\mathrm{I}}_{ijkl}\left(A^{\mathrm{I}}_{klmn}L^{-1}_{mnpq}\dot{\Sigma}_{pq} + H_{klmn}\dot{\varepsilon}^{\mathrm{tr}}_{mn}\right) \tag{42}$$

where

$$H_{ijkl} = A^{\mathrm{I}}_{ijmn}D_{mnkl} + F^{\mathrm{I}}_{ijkl} - I_{ijkl} \tag{43}$$

By the consistency conditions for forward and reverse transformation yielding equations, the proportional factors $\mathrm{d}\lambda$ and $\mathrm{d}\lambda^{\mathrm{re}}$ can be calculated as:

$$\mathrm{d}\lambda = -\frac{\sigma_{ij}^{\mathrm{d(I)}} l_{ijkl}^{\mathrm{I}} A_{klmn}^{\mathrm{I}} L_{mnpq}^{-1} \dot{\Sigma}_{pq}}{\sigma_{ij}^{\mathrm{d(I)}} l_{ijkl}^{\mathrm{I}} H_{klmn} \sigma_{mn}^{\mathrm{d(I)}}} \tag{44}$$

$$\mathrm{d}\lambda^{\mathrm{re}} = -\frac{\sigma_{ij}^{\mathrm{d(I)}} l_{ijkl}^{\mathrm{I}} A_{klmn}^{\mathrm{I}} L_{mnpq}^{-1} \dot{\Sigma}_{pq}}{\sigma_{ij}^{\mathrm{d(I)}} l_{ijkl}^{\mathrm{I}} H_{klmn} \varepsilon_{mn}^{\mathrm{tr}}} \tag{45}$$

The response of the composite under uniaxial tensile loading is calculated for two typical cases. One is ductile aluminum matrix with higher elastic modulus and yield stress than SMA, another case is the soft matrix where both modulus and yield stress are lower than SMA.

The material properties used for SMA are (Lin et al., 1994; 1996; Shaw and Kyriakides, 1995): E = 60 GPa, $\mu = 22.5$ GPa, $\sigma_{\mathrm{e}}^{y} = 100$ MPa and $\sigma_{\mathrm{e}}^{y\mathrm{re}} = 80$ MPa at room temperature, maximum equivalent transformation strain $(\varepsilon_{\mathrm{e}}^{\mathrm{tr}})_{\max} = 6\%$, volume fraction of SMA in composite $f = 20\%$. Macroscopic applied stresses $\Sigma_{33} \neq 0$ and other $\Sigma_{ij} = 0$.

4.1. Case I—aluminum matrix with spherical SMA particulates

The material constants used for alumimum are: Young's modulus E = 70 GPa, $\mu = 27$ GPa, $\sigma_{\mathrm{e}}^{y} = 245$ MPa, h = 85 MPa, n = 0.2 (Hamada et al., 1997). Fig. 3, respectively, shows the individual uniaxial tensile stress–strain curves for the composite, aluminum alone and SMA alone during a uniaxial tensile loading-unloading cycle. From Figs. 3–5, the start and finish of the transformation in SMA and the plastic yielding of matrix can be identified. When SMA inclusions begin to transform while matrix still continues to deform elastically, the deviation of macroscopic response from linearity is quite small. Once the matrix enters plastic deformation the composite exhibits obvious plastic flow and the macroscopic yield stress level of composite is lower than that of matrix alone because of the earlier transformation yielding of SMA. Following this first stage of overall plastic flow the composite begins its second stage hardening (the uprising branch *bc* of the stress strain curve) when transformation is exhausted (SMA returns to elastic response). While this second stage hardening is similar to that of the

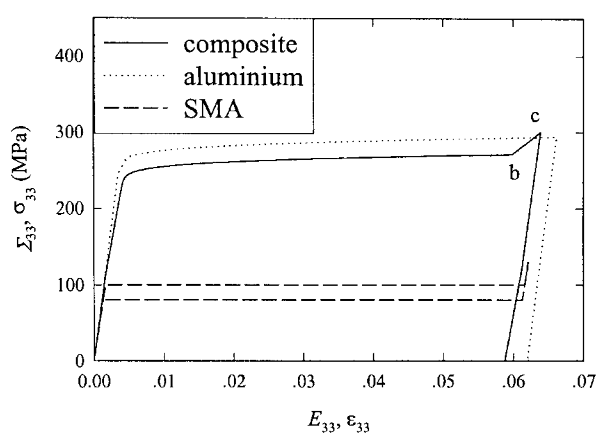

Fig. 3. The axial stress strain curves of composite, aluminum and SMA under uniaxial tension.

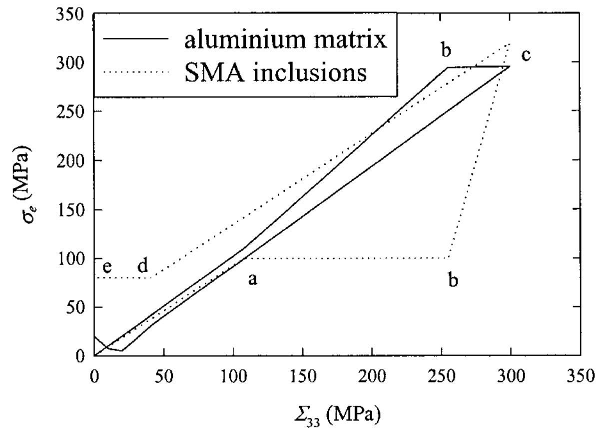

Fig. 4. The evolution of effective stress in aluminum matrix and SMA inclusions.

traditional ductile metal matrix composite reinforced by hard elastic inclusions, it could not be observed in composite with ductile non-SMA metal inclusions. The above two-stage behavior of the SMA composite during loading process is due to the fact that SMA inclusions act like ordinary metal inclusions during transformation and act like elastic inclusions when transformation is finished. Moreover, the results for the non-SMA particulate metal matrix composite can be obtained by assigning a very large value to the transformation strain in the present model.

From Fig. 5 it is seen that a small amount of compressive stress is produced in matrix (about 15 MPa) after unloading, this is due to the partial reverse transformation of SMA during unloading (after unloading most part of the SMA is still martensite). The most significant reinforcement happens in the subsequent heating process where all martensite will transform into parent phase and large amount of compressive stress is produced in the matrix (Hamada et al., 1997; Song et al., 1999). This compressive stress can be relaxed by the forward transformation of SMA, which happens when the composite is cooled. Thus by cooling and heating, the internal stress in both matrix and SMA can be controlled. This is the most distinguished deformation mechanism of SMA composite compared with those of traditional composites.

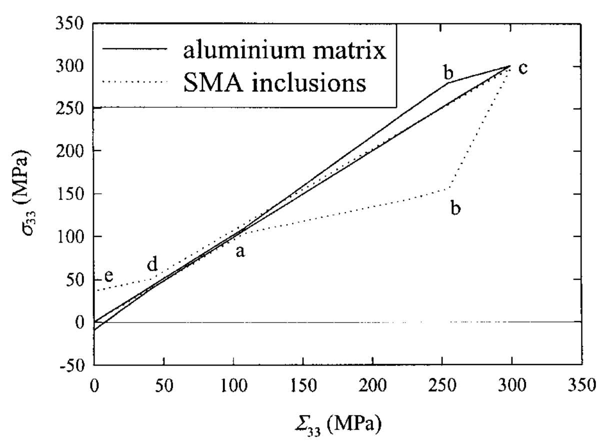

Fig. 5. The evolution of axial stress in aluminum matrix and SMA inclusions.

4.2. Case II—soft matrix with spherical SMA particulates

Contrary to aluminum matrix, here we select another type of ductile materials such as polymers as the matrix whose elastic modulus and yield stress are lower than SMA. The typical material constants for the soft matrix are: Young's modulus $E = 2.9$ GPa, $\mu = 1.115$ GPa, $\sigma_e^y = 20$ MPa, $h = 80$ MPa, $n = 0.5$. Fig. 6, respectively, shows the individual uniaxial tensile stress–strain curves for the composite, matrix alone and SMA alone during a uniaxial tensile loading–unloading cycle. Compared with case I, it is seen that both macroscopic yield stress and elastic modulus of the composite is higher than matrix because harder SMA has strengthening effect on the composite. Similar to case I, during unloading process a compressive axial stress is built up in matrix due to partial reverse transformation (Figs. 7 and 8). Also, the most significant compressive stress in the matrix will be developed by the subsequent heating (Song et al., 1999).

The two examples studied above mainly are concerned with the effect of matrix and SMA properties on the overall performance of the composite. Though the obtained results are limited to the spherical

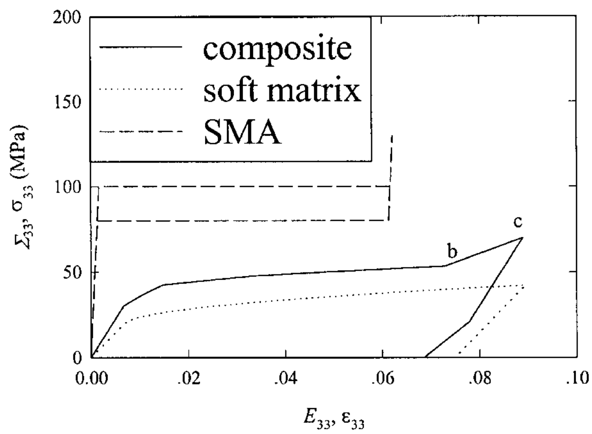

Fig. 6. The axial stress strain curves of composite, soft matrix and SMA under uniaxial tension.

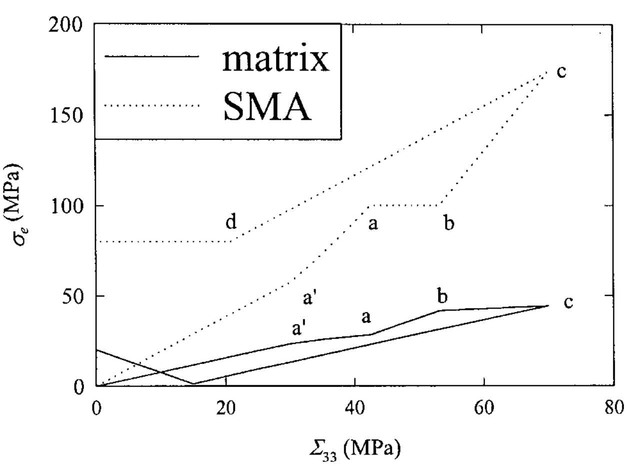

Fig. 7. The evolution of effective stress in soft matrix and SMA inclusions.

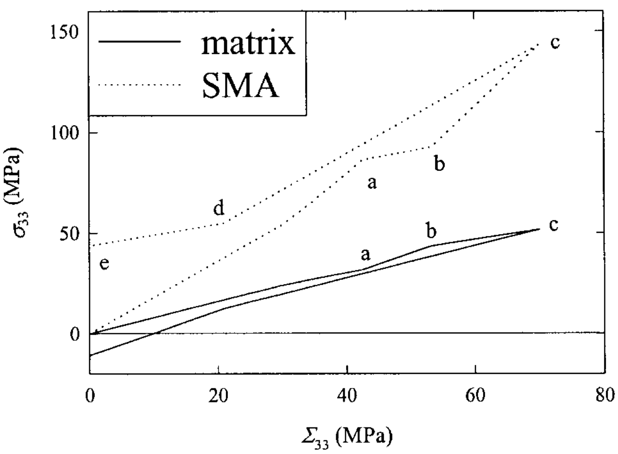

Fig. 8. The evolution of axial stress in soft matrix and SMA inclusions.

shape, the potential of this micromechanics modeling in characterizing the constitutive behavior of the composite and further in microstructure design to obtain a desired material performance has been demonstrated. A systematic study of the effects of temperature and microstructure such as shape, volume fraction and the orientation of SMA inclusions on the macroscopic as well as the internal stress and strain development will be given elsewhere (Song et al., 1999).

5. Conclusions

By using self-consistent approach the constitutive behavior of SMA composite with ductile matrix is investigated. A micromechanics-based quantitative understanding of the role of microstructure and constituent properties in the overall behavior is achieved. Such understanding is based on micro–macro correlations established in this paper and makes it possible to predict the internal stress and strain development in both matrix and SMAs during an externally applied thermomechanical loading process. As a consequence of present research, the microstructure effect such as shape, volume fraction and orientation, mechanical properties of the SMA inclusions and matrix, as well as the effect of applied stress and temperature on the overall behavior of the composite can be quantitatively characterized. The theoretical formulations obtained are general and can be used for both brittle and ductile matrix composite systems. The model is applied to two kinds of composites with spherical SMA particulates. It is believed that the present study can serve as a starting point in the microstructure design of this type of intelligent composites in the future.

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Appendix A. Derivation of the concentration tensor by the self-consistent approach

Under the small strain hypothesis, the governing equations are:

the local constitutive equation

$$\dot{\sigma}_{ij}(r) = l_{ijkl}(r)\dot{\varepsilon}^{\mathrm{ep}}_{kl}(r) \tag{A1}$$

where $\dot{\sigma}(r)$ is the local stress rate, $l(r)$ the local tangent modulus and $\dot{\varepsilon}^{\mathrm{ep}}(r)$ the elastoplastic strain rate which consists of the elastic part $\dot{\varepsilon}^{\mathrm{e}}(r)$ and the plastic part $\dot{\varepsilon}^{\mathrm{p}}(r)$.

Kinematics relations

$$\dot{\varepsilon}_{ij}(r) = \frac{1}{2}\left\{v_{j,i}(r) + v_{j,i}(r)\right\} \tag{A2}$$

where $\dot{\varepsilon}(r)$ is the total strain rate and v_i the particle velocity. With small strain hypothesis, it follows that

$$\dot{\varepsilon}_{ij}(r) = \dot{\varepsilon}^{\mathrm{ep}}_{ij}(r) + \dot{\varepsilon}^{*}_{ij}(r) \tag{A3}$$

$\dot{\varepsilon}^{*}(r)$ is the eigen strain rate due to martensitic phase transformation in SMA and etc.

Quasi-static equilibrium equation (no body forces)

$$\dot{\sigma}_{ij,j}(r) = 0 \tag{A4}$$

From the constitutive relation (A1) and the decomposition (A3) of the total strain, eqn (A4) is equivalent to

$$\left[l_{ijkl}(r)\left(\dot{\varepsilon}_{kl}(r) - \dot{\varepsilon}^{*}_{kl}(r)\right)\right]_{,j} = 0 \tag{A5}$$

Using the usual symmetries of $l(r)$, one can write eqn (A5) as

$$\left[l_{ijkl}(r)\left(v_{k,l}(r) - \dot{\varepsilon}^{*}_{kl}(r)\right)\right]_{,j} = 0 \tag{A6}$$

Another form of eqn (A6) can be obtained by introducing a reference homogeneous medium with tangent modulus L^0 and which undergoes an uniform eigen strain rate $\dot{E}^{*0}$ such that

$$l_{ijkl}(r) = L^0_{ijkl} + \delta l_{ijkl}(r) \tag{A7}$$

$$\dot{\varepsilon}^{*}_{ij}(r) = \dot{E}^{*0}_{ij} + \delta\dot{\varepsilon}^{*}_{ij}(r) \tag{A8}$$

where $\delta l(r)$ and $\delta\dot{\varepsilon}^{*}(r)$ are the corresponding fluctuations with respect to the reference homogeneous medium. Substituting eqns (A7) and (A8) into eqn (A6) and using the property $(L^0_{ijkl}\dot{E}^{*0}_{kl})_{,j} = 0$, one obtains

$$L^0_{ijkl}v_{k,lj}(r) + \left[\delta l_{ijkl}(r)\dot{\varepsilon}^{\mathrm{ep}}_{kl}(r) - L^0_{ijkl}\delta\dot{\varepsilon}^{*}_{kl}(r)\right]_{,j} = 0 \tag{A9}$$

which is equivalent to the Lame equations of homogeneous non-linear problem with body force rate

$$\dot{f}_i(r) = \left[\delta l_{ijkl}(r)\dot{\varepsilon}^{\mathrm{ep}}_{kl}(r) - L^0_{ijkl}\delta\dot{\varepsilon}^{*}_{kl}(r)\right]_{,j} \tag{A10}$$

eqn (A9) can be transformed into an integral equation using the Green tensor G^0 of an homogeneous infinite medium with tangent modulus L^0 defined by

$$L^0_{ijkl}G^0_{jn,ik}(r-r')+\delta_{ln}\delta(r-r')=0 \tag{A11}$$

After some algebra calculations, one gets

$$v_{m,n}(r)=v^0_{m,n}+\int_{V'}G^0_{mj,in}(r-r')\Big[\delta l_{ijkl}(r')\dot{\varepsilon}^{\mathrm{ep}}_{kl}(r')-L^0_{ijkl}\delta\dot{\varepsilon}^*_{kl}(r')\Big]\,\mathrm{d}V' \tag{A12}$$

From eqn (A2), one obtains the local strain rate

$$\dot{\varepsilon}_{ij}(r)=\dot{E}_{ij}-\int_{V'}\Gamma^0_{ijkl}(r-r')\big[\delta l_{klmn}(r')\dot{\varepsilon}^{\mathrm{ep}}_{mn}(r')-L^0_{klmn}\delta\dot{\varepsilon}^*_{mn}(r')\big]\,\mathrm{d}V' \tag{A13}$$

where Γ^0 is the modified Green tensor defined by

$$\Gamma^0_{ijkl}(r-r')=-\frac{1}{2}\Big[G^0_{ki,jl}(r-r')+G^0_{li,jk}(r-r')\Big] \tag{A14}$$

which can be decomposed into a local Γ^l and a non local part Γ^{nl} such that

$$\Gamma^0_{ijkl}(r-r')=\Gamma^l_{ijkl}\delta(r-r')+\Gamma^{nl}_{ijkl}(r-r') \tag{A15}$$

with eqn (A15), eqn (A13) becomes

$$\dot{\varepsilon}_{ij}(r)=\dot{E}_{ij}-\Gamma^l_{ijkl}(r)\big[\delta l_{klmn}(r)\dot{\varepsilon}^{\mathrm{ep}}_{mn}(r)-L^0_{klmn}\delta\dot{\varepsilon}^*_{mn}(r)\big]-\int_{V'}\Gamma^{nl}_{ijkl}(r-r')F_{kl}(r')\,\mathrm{d}V' \tag{A16}$$

where

$$F_{kl}(r)=\delta l_{klmn}(r)\dot{\varepsilon}^{\mathrm{ep}}_{mn}(r)-L^0_{klmn}\delta\dot{\varepsilon}^*_{mn}(r) \tag{A17}$$

In general the integral term in eqn (A16) is very difficult to calculate due to the fluctuation of the field $F(r)$. The original idea of the self-consistent scheme is to choose the reference medium, defined by L^0 and $\dot{E}^{*0}$, so that the mean value of $F(r)$ over V is zero. In other words, this allows neglecting the integral term in eqn (A16) compared with the local term. That is

$$\int_V\big[\delta l_{klmn}(r)\dot{\varepsilon}^{\mathrm{ep}}_{mn}(r)-L^0_{klmn}\delta\dot{\varepsilon}^*_{mn}(r)\big]\,\mathrm{d}V=0 \tag{A18}$$

or

$$\int_V\Big[\big(l_{klmn}(r)-L^0_{klmn}\big)\dot{\varepsilon}^{\mathrm{ep}}_{mn}(r)-L^0_{klmn}\Big(\dot{\varepsilon}^*_{mn}(r)-\dot{E}^{*0}_{mn}\Big)\Big]\,\mathrm{d}V=0 \tag{A19}$$

Using the fact that

$$\dot{\Sigma}=\frac{1}{V}\int_V\dot{\sigma}(r)\,\mathrm{d}V\quad\text{and}\quad\dot{E}=\frac{1}{V}\int_V\dot{\varepsilon}(r)\,\mathrm{d}V \tag{A20}$$

eqn (A19) is equivalent to

$$\dot{\Sigma}_{ij} = L^0_{ijkl}\left(\dot{E}_{kl} - \dot{E}^{*0}_{kl}\right) \tag{A21}$$

Since the overall behavior of the composite is given by

$$\dot{\Sigma}_{ij} = L_{ijkl}\left(\dot{E}_{kl} - \dot{E}^{*}_{kl}\right) \tag{A22}$$

We see that the self-consistent approximation requires to choose L^0 and $\dot{E}^{*0}$ so that $L^0 = L$ and $\dot{E}^{*0} = \dot{E}^*$. Under such conditions, eqn (A16) becomes

$$\dot{\varepsilon}_{ij}(r) = \dot{E}_{ij} - \Gamma^l_{ijkl}(r)\left[\delta l_{klmn}(r)\dot{\varepsilon}^{\mathrm{ep}}_{mn}(r) - L_{klmn}\delta\dot{\varepsilon}^*_{mn}(r)\right] \tag{A23}$$

where

$$\delta l(r) = l(r) - L \tag{A24}$$

$$\delta\dot{\varepsilon}^*(r) = \dot{\varepsilon}^*(r) - \dot{E}^* \tag{A25}$$

and the local part of the modified Green tensor Γ^l is related to the usual Eshelby tensor S_{ijkl} through the following relation

$$\Gamma^l_{ijkl}(r) = S_{ijmn}(r)L^{-1}_{mnkl} \tag{A26}$$

eqn (A23) can be written formally as the form of eqn (A13)

$$\dot{\varepsilon}_{ij}(r) = A_{ijkl}(r)\dot{E}_{kl} + \dot{a}_{ij}(r) \tag{A27}$$

from which the concentration tensors $A(r)$ and $\dot{a}_{ij}(r)$ can be deduced as

$$A_{ijkl}(r) = \left[I_{ijkl} + S_{ijmn}(r)L^{-1}_{mnpq}\delta l_{pqkl}(r)\right]^{-1} \tag{A28}$$

$$\dot{a}_{ij}(r) = A_{ijkl}(r)S_{klmn}(r)L^{-1}_{mnpq}\left[l_{pqrs}(r)\dot{\varepsilon}^*_{rs}(r) - L_{pqrs}\dot{E}^*_{rs}\right] \tag{A29}$$

From eqns (A28) and (A29) the average strain concentration tensors are easily obtained as

$$\langle A_{ijkl}(r)\rangle_{V^{\mathrm{I}}} = \left\langle\left[I_{ijkl} + S_{ijmn}(r)L^{-1}_{mnpq}\left(l^{\mathrm{I}}_{pqkl} - L_{pqkl}\right)\right]^{-1}\right\rangle_{V^{\mathrm{I}}} \tag{A30}$$

$$\langle A_{ijkl}(r)\rangle_{V^{\mathrm{M}}} = \left\langle\left[I_{ijkl} + S_{ijmn}(r)L^{-1}_{mnpq}\left(l^{\mathrm{M}}_{pqkl} - L_{pqkl}\right)\right]^{-1}\right\rangle_{V^{\mathrm{M}}} \tag{A31}$$

$$\langle a_{ij}(r)\rangle_{V^{\mathrm{I}}} = \left\langle A_{ijkl}(r)S_{klmn}(r)L^{-1}_{mnpq}\left[l^{\mathrm{I}}_{pqrs}\dot{\varepsilon}^*_{rs}(r) - L_{pqrs}\dot{E}^*_{rs}\right]\right\rangle_{V^{\mathrm{I}}} \tag{A32}$$

$$\langle a_{ij}(r)\rangle_{V^{\mathrm{M}}} = -\langle A_{ijkl}(r)S_{klmn}(r)\rangle_{V^{\mathrm{M}}}\dot{E}^*_{mn} \tag{A33}$$

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